SYNOPTIC: Motion of a Symmetric Gyrostat in a Viscous Medium, T. R. Kane, Stanford University, Stanford, Calif.; *AIAA Journal*, Vol. 8, No. 10, pp. 1786–1789.

Spacecraft Technology; Spacecraft Dynamics and Control

Theme

Analytical study of attitude motions of a gyrostat in a resisting medium.

Content

The system under consideration is a gyrostat G consisting of two rigid bodies, A and B. The central inertia ellipsoids of B and G are ellipsoids of revolution with parallel axes; the mass center of B is fixed in A; and B can rotate relative to A about its symmetry axis.

The dynamical equations for this system can be cast into the form

$$I\dot{p}_j = \mathbf{R}\cdot\mathbf{c}_j, \quad j = 1,2,3$$

$$\dot{r} = [J/K(J-K)]\mathbf{c}_3\cdot\mathbf{M} - [\mathbf{R}\cdot\mathbf{c}_3/(J-K)]$$

where I, J, and K are moments of inertia; $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are unit vectors, the last of these being parallel to the symmetry axis of G; p_1, p_2, and p_3 are angular velocity components of a reference frame in which $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are fixed and in which A rotates with a rate $s = [(I-J)p_3 - Kr]/J$; r is the angular speed of B relative to A; $\mathbf{R}$ is the moment about the mass center of G of all forces applied to A by the resisting medium in which A moves; and $\mathbf{M}$ is the moment about the mass center of B of all forces exerted by A on B.

Taking

$$\mathbf{R} = -\beta I(p_1\mathbf{c}_1 + p_2\mathbf{c}_2) - \gamma(I/J)(Ip_3 - Kr)\mathbf{c}_3$$

where β and γ are constants, one can integrate the equations of motion, both for the case when B is completely free to rotate relative to $A(\mathbf{c}_3\cdot\mathbf{M} = 0)$ and for a rotor driven at constant angular speed ($\dot{r} = 0$); and a perturbation technique can then be used to solve three kinematical equations of the form

$$\dot{n}_1 = n_2p_3 - n_3p_2$$

where $n = \mathbf{n}\cdot\mathbf{c}_2$ and $\mathbf{n}$ is an inertially fixed unit vector which is equal to $\mathbf{c}_3$ at $t = 0$. The results of this integration permit one to find the limiting angle θ^* between $\mathbf{n}$ and the axis of symmetry of G:

$$\theta^* = |a_2(0)|\{\beta^2 + [(K/I)r(0)]^2\}^{-1/2}$$

where $|a_2(0)|$ is the magnitude of the initial transverse angular velocity of A. This formula yields values in good agreement with those resulting from a numerical integration of the kinematical equations, provided ϵ, defined as $\epsilon = a_2(0)/\beta$, is sufficiently small.

Motion of a Symmetric Gyrostat in a Viscous Medium

T. R. Kane*
Stanford University, Stanford, Calif.

Dynamical equations governing the motion of a symmetric gyrostat in a viscous medium are solved in closed form, both for the case of a free rotor and for a rotor driven with constant angular speed. Kinematical equations of the Poisson type are then solved by a perturbation technique to obtain a formula for the limiting angle between the symmetry axis of the gyrostat and the fixed line with which this axis coincides initially; and this formula is tested by means of computer solutions of the kinematical equations.

Introduction

In the more than sixty years that have elapsed since the publication of Gray's comprehensive work on gyrostats,[1] two major developments bearing directly on this subject have occurred: dual spin satellites have begun to play an important role in the field of space flight, and highly effective computing machines have come into wide use. The first of these suggests that fundamental insights may have more practical significance now than ever before, and the second means that one can employ a powerful, previously unavailable tool to gain such insights. With these ideas in mind, it was decided to seek information relevant to the following fundamental question: How does a dual spin satellite behave in a resisting medium? What follows is an attempt to provide a partial answer to this question by solving a certain problem in dynamics.

Received December 8, 1969; revision received March 13, 1970. This work was supported by NASA, under NGR-05-020-209.

* Professor of Applied Mechanics.

Analysis

The system to be analyzed is a gyrostat G (Fig. 1) consisting of two rigid bodies, A and B. The inertia ellipsoid of A for the mass center A^* of A may have three unequal principal diameters, whereas the inertia ellipsoid E_B of B for the mass center B^* of B is presumed to be an ellipsoid of revolution. Furthermore, B is connected to A in such a way that (1) B^* and the axis of revolution of E_B are fixed in A, but B can rotate relative to A about this axis, and (2) the inertia ellipsoid E_G of the gyrostat G for the mass center G^* of G is an ellipsoid of revolution whose axis is parallel to that of E_B. Torque-free motions of precisely this system were discussed in a previous paper[2] in this Journal, and, for the sake of brevity, information contained in that paper is used wherever possible in the sequel.

Before choosing a mathematical description for the action of the resisting medium on body A, it is convenient to introduce certain inertial and kinematical parameters.

In Fig. 1, $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ designate mutually perpendicular unit vectors oriented in such a way that $\mathbf{c}_3$ is parallel to the axes of